CSE383M and CS395T Surprise Quiz

April 10, 2013

This test is closed notes, books, and computers.

The distribution Exponential(β) has the pdf $p(x) = \beta e^{-\beta x}$, with $0 \leq x < \infty$. (I'm sure you knew that!)

1. (10 points) An value x is chosen with probability c from Exponential(β_1), otherwise from Exponential(β_2). What is x's pdf, $p(x)$?

2. (15 points) For $i = 1, \ldots, N$, you are given a set of probabilities p_i with $\sum_i p_i = 1$, and a set of positive values β_i. A value of i is drawn with probability p_i, and a value x is then drawn from Exponential(β_i). Write an expression for x's pdf, $p(x)$. If you use any sum signs, be sure to include their lower and upper limits.

3. (25 points) A value β is drawn from the distribution Exponential(α). A value x is then drawn from the distribution Exponential(β). What is x's pdf, $p(x)$?

Hints: (i) Your answer should not contain the symbol "β". (ii) Here is a possibly useful definite integral:

$$\int_0^\infty r e^{-\lambda r} dr = \frac{1}{\lambda^2}$$

Bill's Solution Set

Quiz 4/10/13

1. It's a mixture of the 2 pdfs with weights c and $(1-c)$:

$$f(x) = c\beta_1 e^{-\beta_1 x} + (1-c)\beta_2 e^{-\beta_2 x}, \quad x > 0$$

2. It's a mixture of N pdfs with weights p_i

$$f(x) = \sum_{i=1}^{N} p_i \underbrace{\beta_i e^{-\beta_i x}}, \quad x > 0$$

3. This is the fun problem. It's a continuous mixture! Since β is drawn from Exponential(α) it has weights $\alpha e^{-\alpha\beta} d\beta$ and the sum in problem 2 becomes an integral. So

$$f(x) = \int_0^\infty \alpha e^{-\alpha\beta} \underbrace{\beta e^{-\beta x}} d\beta$$

this is like the underlined piece in prob. 2

$$= \alpha \int_0^\infty \beta e^{-\beta(\alpha+x)} d\beta$$

see problem hint for the integral

$$= \frac{\alpha}{(\alpha+x)^2}, \quad x > 0$$

(I then checked that the integral of this $\int dx$ is 1, but you don't have to.)