

CSE383M and CS395T Surprise Quiz

March 4, 2013

For the following problems, consider the univariate distribution

$$p_X(x) = \begin{cases} 1 - \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

1. Sketch the distribution $p_X(x)$.
2. What is the distribution's mean and standard deviation?
3. What is its cumulative distribution function (CDF)?
4. Write code or pseudocode for drawing random deviates from the distribution. (You may assume that you have a random generator for uniform $(0, 1)$.)
5. What is the approximate distribution of the sum S of N deviates drawn from $p_X(x)$, where $N \gg 1$? (Write down an explicit function of S and N .)
6. You are given 28 values x_i in the range $0 < x_i < 2$. Describe in detail how you might use S from problem 5 as a test statistic for deciding whether these values are drawn i.i.d. from $p_X(x)$. (You can give your answer either in words, pseudocode, or code.)
7. How would your answer to problem 6 be different if $p_X(x)$ was one of 10 different hypotheses, each with its own known distribution $p_X^{(j)}(x)$ with $(j) = 1, 2, \dots, 10$, and you want to know which hypothesis is correct? (Here (j) is a label, not an exponent!)
8. What would be a Bayesian approach to problem 7? Write down an explicit test statistic and indicate how to use it.
9. What is the distribution of the sum $z = x_1 + x_2$ where x_1 and x_2 are drawn from $p_X(x)$? If possible, compute the distribution function $p_{X_1+X_2}(z)$ explicitly. (This problem, although perfectly do-able, is just to waste your time if you have finished all the others.)