

Surprise Quiz for 1/27/14

1. There are two caves, A and B. Cave A contains 7 red kangaroos and 2 blue kangaroos. Cave B contains 3 red kangaroos and 5 blue kangaroos. You pick a cave at random. One kangaroo, randomly selected, hops out of it.

1(a). What is the probability that the kangaroo that hops out is blue?

1(b). If it is blue, what is the probability that it came from Cave B?

$$(a) P(\text{blue}) = P(\text{blue}|A)P(A) + P(\text{blue}|B)P(B)$$

$$= \frac{2}{2+7} \cdot \frac{1}{2} + \frac{5}{5+3} \cdot \frac{1}{2} = \frac{1}{9} + \frac{5}{16} = \frac{61}{144} \approx 0.42$$

$$(b) P(B|\text{data}) = \frac{P(\text{data}|B)P(B)}{P(\text{data}|A)P(A) + P(\text{data}|B)P(B)}$$

$$= \frac{\frac{5}{5+3} \cdot \frac{1}{2}}{\frac{2}{2+7} \cdot \frac{1}{2} + \frac{5}{5+3} \cdot \frac{1}{2}} = \frac{\frac{5}{16}}{\frac{61}{144}} = \frac{45}{61} \approx 0.74$$

2. An event Q that depends on some unknown parameter x (with $0 \leq x \leq 2$) occurs with probability

$$P(Q|x) = \frac{1}{7}x^2$$

2(a). Is it true that $\int_0^2 P(Q|x)dx = 1$? If not, is this a problem?

2(b). While you don't know the value of x , you do have a prior on its distribution. If your prior is uniform in x , what is the probability $P(Q)$, that is, marginalized over x ? Hint: The answer is a number, not a function of anything.

2(c). Same as 2(b), but your prior is instead a massed prior at $x = 1$. What is $P(Q)$ in this case?

$$(a) \int_0^2 \frac{1}{7}x^2 dx = \left. \frac{1}{7} \cdot \frac{1}{3} x^3 \right|_0^2 = \frac{8}{21} \neq 1$$

Not a problem since there is not necessarily any relation between $P(Q|x)$ for different x 's, as long as all values are $0 \leq P(Q|x) \leq 1$. (It's a probability of Q , not x !)

$$(b) \int_0^2 P(Q|x)P(x)dx = \int_0^2 \left(\frac{1}{7}x^2\right)\left(\frac{1}{2}\right)dx = \frac{8}{21} \cdot \frac{1}{2} = \frac{4}{21} \approx 0.19$$

$$(c) \int_0^2 P(Q|x)\delta(x-1)dx = \left. \frac{1}{7}x^2 \right|_{x=1} = \frac{1}{7} \approx 0.14$$