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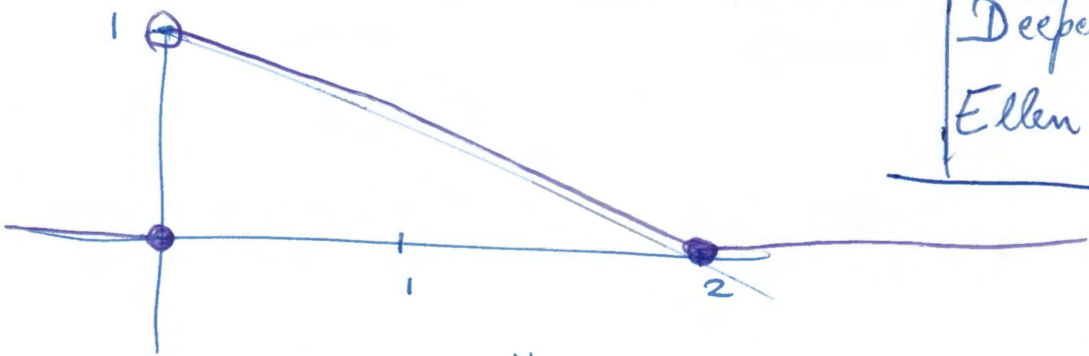
1 $$p_X(x) = \begin{cases} 1-\frac{x}{2} & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$



2. $$\langle x \rangle = \int_0^2 x\left(1-\frac{x}{2}\right)dx = \int_0^2 \left(x - \frac{x^2}{2}\right)dx$$

$$= \frac{1}{2}x^2 - \frac{x^3}{6}\Big|_0^2 = 2 - \frac{8}{6} = \frac{12}{6} - \frac{8}{6} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

mean

$$\langle x^2 \rangle = \int_0^2 x^2\left(1-\frac{x}{2}\right)dx = \int_0^2 \left(x^2 - \frac{x^3}{2}\right)dx$$

$$= \frac{1}{3}x^3 - \frac{x^4}{8}\Big|_0^2 = \frac{8}{3} - \frac{16}{8} = \frac{8}{3} - \frac{6}{3} = \frac{2}{3}$$

$$\text{var}(x) = \langle x^2 \rangle - \langle x \rangle^2 = \left(\frac{2}{3}\right) - \left(\frac{2}{3}\right)^2 = \boxed{\frac{2}{9}}$$

$$\boxed{\sigma = \sqrt{\text{var}(x)} = \frac{1}{3}\sqrt{2}}$$

3. $$F(x) = \int_0^x \frac{dF}{dx}\,dy = \int_0^x p_X(y)\,dy$$

$$= \int_0^x \left(1-\frac{y}{2}\right)dy = y - \frac{y^2}{4}\Big|_0^x = x - \frac{x^2}{4}$$

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$$F(x) = x - \frac{x^2}{4}$$

$$= -\left(\frac{x^2}{4} - x + 1\right) + 1$$

$$= -\left(\frac{x}{2} - 1\right)^2 + 1$$

$$\frac{x}{2} - 1 = s(1-F)^{\frac{1}{2}} \quad \text{where } s = \begin{cases} +1 \\ -1 \end{cases}$$

$$x = 2\left\{ s(1-F)^{\frac{1}{2}} + 1 \right\}$$

$F = 0, \; s = +1 \implies x = 2 \cdot 2 = 4$

$F = 1, \; s = +1 \implies x = 2$

$F = 0, \; s = -1 \implies x = 0$

$F = 1, \; s = -1 \implies x = 2$

Hence, we choos $s = -1$, such that $x \in [0, 2]$.

pseudo code

function x = RandD( )

% draw random number between 0 and 1

F = rand;

% compute random deviate x

$x = -2\sqrt{1-F} + 2$

return x

5] By the Central limit theorem,

The random variable $S = \sum_{i=1}^{N} X_i$ is approximately Gaussian distributed.

We have as expectation values of the mean and variance;

$$E[S] = \sum_{i=1}^{N} E[X_i] = N \cdot \mu_{X} = N \cdot \frac{2}{3}$$

$$E\left[(S - N\cdot\mu_X)^2\right] = E\left[\left(\sum_{i=1}^{N} (X_i - \mu_X)\right)^2\right]$$

$$= \sum_{i=1}^{N} E\left[(X_i - \mu_X)^2\right]$$

$$= N \cdot \sigma_X^2$$

$$= N \cdot \frac{2}{9}$$

<u>Note</u>

The expectation of the sum is equal to the sum of the expectations, because we consider independent random variables X_i, for $i = 1, 2, \dots N$.

6 (We compare the sample mean μ_{S^*} and the sample variance $\operatorname{Var}[S^*]$ directly with that obtained in question 5.

$$S^* = \sum_{i=1}^{28} X_i$$

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H_i : random variable X_i with distribution $P_{X_i}(x)$ with the mean μ_i and variance σ_i^2 for $i = 1, 2, \ldots 10$.

Use maximum likely hood approach to determine the most likely values of the parameters to generate the data.

8| Applying MAP estimate:

Bayes rule: choose

$$\max_i \frac{P(\text{Data} \mid H_i)\, P(H_i)}{\sum_j P(\text{Data} \mid H_j)\, P(H_j)}$$

9. Let z be the sum of x, & y which are drawn from the distribution p_X.

$$\boxed{z = x + y}$$

Then, z is distributed as,

$$p_Z(z) = \int_{\max(0,z-2)}^{\min(2,z)} p_X(x)\, p_X(z-x)\, dx$$

$$= \int_{\max(0,z-2)}^{\min(2,z)} \left(1 - \frac{x}{2}\right)\left(1 - \left(\frac{z-x}{2}\right)\right) dx$$

$$= \left(1 - \frac{z}{2}\right)x + \frac{x^2 z}{8} - \frac{x^3}{12} \Bigg|_{\max(0,z-2)}^{\min(2,z)}$$

$$p_Z(z) = \begin{cases} \frac{z^3}{24} + z - \frac{z^2}{2}, & z \in [0,2] \\ \frac{8}{3} - 2z - \frac{z^3}{24} + \frac{z^2}{2}, & z \in (2,4] \end{cases}$$