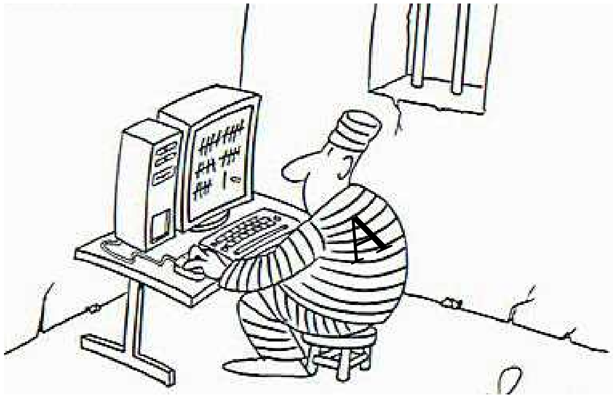
Opinionated Lessons in Statistics

by Bill Press

#4 The Jailer's Tip

Our next topic is Bayesian Estimation of Parameters. We'll ease into it with an example that looks a lot like the Monte Hall Problem:

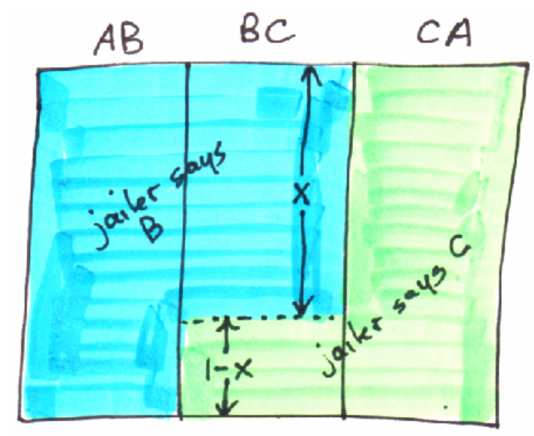


The Jailer's Tip:

- Of 3 prisoners (A,B,C), 2 will be released tomorrow.
- A, who thinks he has a 2/3 chance of being released, asks jailer for name of one of the lucky – but not himself.
- Jailer says, truthfully, "B".
- "Darn," thinks A, "now my chances are only ½, C or me".

Is this like Monty Hall? Did the data ("B") change the probabilities?

Further, suppose (unlike Monty Hall) the jailer is not indifferent about responding "B" versus "C". Does that change your answer to the previous question?



$$P(S_B|BC) = x, \quad (0 \le x \le 1)$$

"says B"

$$\begin{aligned} P(A|S_B) &= P(AB|S_B) + P(AC|S_B) \\ &= \frac{P(S_B|AB)P(AB)}{P(S_B|AB)P(AB) + P(S_B|BC)P(BC) + P(S_B|CA)P(CA)} \\ &= \frac{\frac{1}{3}}{1\cdot\frac{1}{3} + x\cdot\frac{1}{3} + 0} = \frac{1}{1+x} \end{aligned}$$

0 1 1/3 x

So if A knows the value x, he can calculate his chances.
If *x*=1/2 (like Monty Hall), his chances are 2/3, same as before; so (unlike Monty Hall) he got no new information.
If *x*≠1/2, he does get new info – his chances change.

But what if he doesn't know *x* at all?

"Marginalization" (this is important!)

- When a model has unknown, or uninteresting, parameters we "integrate them out" …
- …multiplying by any knowledge of their distribution
  - At worst, just a prior informed by background information
  - At best, a narrower distribution based on data
- This is not any new assumption about the world
  - it's just the Law of de-Anding

(e.g., Jailer's Tip):

law of de-Anding:
x's are EME!

$$P(A|S_B I) = \int_x P(A|S_B x I)\, p(x|I)\, dx$$
$$= \int_x \frac{1}{1+x}\, p(x|I)\, dx$$

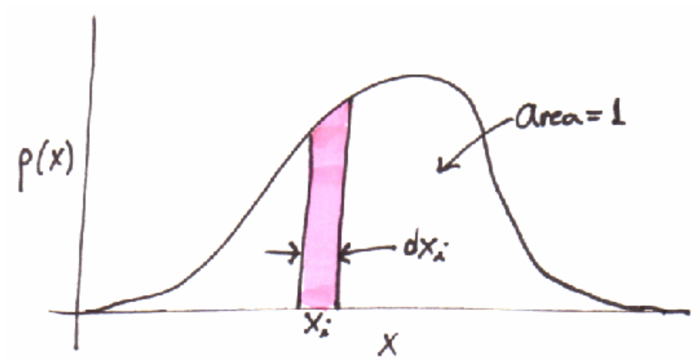
(repeating previous equation:)

$$P(A|S_B I) = \int_x P(A|S_B x I)\, p(x|I)\, dx$$
$$= \int_x \frac{1}{1+x}\, p(x|I)\, dx$$

first time we've seen a *continuous* probability distribution, but we'll skip the obvious repetition of all the previous laws

$$p(x) \equiv p(x|I)$$

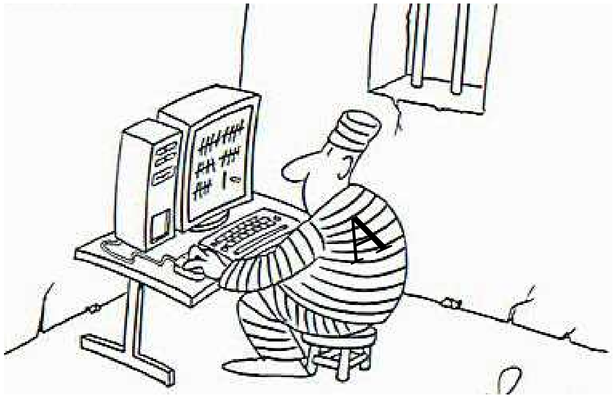
(Notice that p(x) is a probability of a probability! That is fairly common in Bayesian inference.)



$$\sum_i P_i = 1 \;\leftrightarrow\; \sum_i p(x_i) dx_i = 1 \;\leftrightarrow\; \int_x p(x)\, dx = 1$$

(repeating previous equation:)

$$P(A|S_B I) = \int_x P(A|S_B x I)\, p(x|I)\, dx$$
$$= \int_x \frac{1}{1+x}\, p(x|I)\, dx$$



What should Prisoner A take for *p(x)* ?
Maybe the "uniform prior"?

$$p(x) = 1, \quad (0 \le x \le 1)$$
$$P(A|S_B I) = \int_0^1 \frac{1}{1+x} dx = \ln 2 = 0.693$$

Not the same as the "massed prior at x=1/2"

$$p(x) = \delta(x - \tfrac{1}{2}), \quad (0 \le x \le 1)$$
$$P(A|S_B I) = \frac{1}{1+1/2} = 2/3$$

"Dirac delta function"

substitute value and remove integral